

# 1-b Approximation Method for $x^n + bx + c = 0$

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June 4, 2002

In this paper, I propose a iteration method to approximate one solution to high rank polynomial equation in the form of  $x^n + bx + c = 0$ .

## 1-b Method

The iteration formula I propose is the following:

$$\pm \sqrt[n-1]{\frac{-c}{\pm \sqrt[n-1]{\frac{-c}{\pm \sqrt[n-1]{1-b}} - b}} - b}.$$

In this way, if we iterate many times, this value will converge to one exact solution of the equation.

For even  $n$ , always choose  $+$ .

For an odd  $n$ , if  $c$  is negative, choose  $+$ , otherwise choose  $-$ . For odd  $n$ , the method might not work. To determine which equation works, the coefficients of the equation  $x^n + bx + c = 0$  must make the following two formula meaningful and one iteration root lies between

$$\sqrt[n-1]{\frac{-c}{\sqrt[n-1]{\frac{-c}{\sqrt[n-1]{1-b}} - b}} - b}$$

and

$$\sqrt[n-1]{\frac{-c}{\sqrt[n-1]{1-b}} - b}.$$

## How it was derived?

For  $x^n + bx + c = y$ , let  $y = x + c$ , we get  $x = \sqrt[n-1]{1-b}$ ; then  $(\sqrt[n-1]{1-b})x + c = 0$ , we get  $x = \frac{-c}{\sqrt[n-1]{1-b}}$ ;

Now  $x^n + bx + c = \frac{-c}{\sqrt[n-1]{1-b}}x + c$ , we get  $x = \sqrt[n-1]{\frac{-c}{\sqrt[n-1]{1-b}} - b}$ .

Repeat this.

## Examples:

1.  $x^5 - x + 1 = 0$ .

Estimate  $-\sqrt[4]{\frac{1}{-\sqrt[4]{2}}} + 1$ , approximate -1.1676, which makes the equation -0.002453.

2.  $x^8 + 20x + 2 = 0$ .

Estimate  $\sqrt[7]{\frac{-2}{\sqrt[7]{-19}}} - 20$ , approximate -1.51927, which makes the equation -0.000095095.

3.  $x^{20} + 500x - 8 = 0$ .

Estimate  $\sqrt[19]{\frac{8}{\sqrt[19]{1-500}}} - 500$ , approximate -1.38775, which makes the equation 0.0000341368.

4.  $x^3 - 100x + 0.1 = 0$ .

Estimate  $\sqrt{\frac{-0.1}{-\sqrt{1+100}}} + 100$ , approximate -10.00049751, which makes the equation -0.000000501.

Repeat the iteration process until you are satisfied with your approximation. I have tried many examples and found that this method most time gives a reasonable estimation.