1-b Approximation Method for $x^n + bx + c = 0$

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In this paper, I propose a iteration method to approximate one solution to high rank polynomial equation in the form of $x^n + bx + c = 0$.

1-b Method

The iteration formula I propose is the following:

$$\pm \sqrt[n-1]{\frac{-c}{\pm \sqrt[n-1]{\frac{-c}{\pm \sqrt[n-1]{1-b}} - b}} - b}.$$

In this way, if we iterate many times, this value will converge to one exact solution of the equation.

For even n, always choose $+$.

For an odd n, if c is negative, choose $+$, otherwise choose $-$. For odd n, the method might not work. To determine which equation works, the coefficients of the equation $x^n + bx + c = 0$ must make the following two formula meaningful and one iteration root lies between

$$\sqrt[n-1]{\frac{-c}{\sqrt[n-1]{\frac{-c}{\sqrt[n-1]{1-b}} - b}} - b}$$

and

$$\sqrt[n-1]{\frac{-c}{\sqrt[n-1]{1-b}} - b}.$$

How it was derived?

For $x^n+bx+c=y$, let $y=x+c$, we get $x=\sqrt[n-1]{1-b}$; then $(\sqrt[n-1]{1-b})x+c=0$, we get $x=\frac{-c}{\sqrt[n-1]{1-b}}$;

Now $x^n+bx+c=\frac{-c}{\sqrt[n-1]{1-b}}x+c$, we get $x=\sqrt[n-1]{\frac{-c}{\sqrt[n-1]{1-b}}-b}$.

Repeat this.

Examples:

1. $x^5-x+1=0$.

   Estimate $-\sqrt[4]{\frac{1}{-\sqrt[4]{2}}+1}$, approximate -1.1676, which makes the equation -0.002453.

2. $x^8+20x+2=0$.

   Estimate $\sqrt[7]{\frac{-2}{\sqrt[7]{-19}}-20}$, approximate -1.51927, which makes the equation -0.000095095.

3. $x^{20}+500x-8=0$.

   Estimate $\sqrt[19]{\frac{8}{\sqrt[19]{1-500}}-500}$, approximate -1.38775, which makes the equation 0.0000341368.

4. $x^3-100x+0.1=0$.

   Estimate $\sqrt{\frac{-0.1}{-\sqrt{1+100}}+100}$, approximate -10.00049751, which makes the equation -0.000000501.

Repeat the iteration process until you are satisfied with your approximation. I have tried many examples and found that this method most time gives a reasonable estimation.